THE SERBIAN SCHOOL OF MATHEMATICS – FROM MIHAILO PETROVIĆ TO THE SHANGHAI LIST

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In 19th-century Serbia, there were six doctors of mathematical sciences. One of them was Mihailo Petrović[34], the founder of Belgrade School of Mathematics. Petrović and his successors–disciples contributed to the development of mathematical education in Belgrade, in towns and cities of the former Yugoslavia: Skopje, Sarajevo, Banja Luka, Zagreb, Podgorica, and particularly in university centres in Serbia: Novi Sad, Niš and Kragujevac. They contributed to the development of mathematics, each in his own way: through scientific work, education of young generations, excellent teaching, writing of textbooks, etc. In this text, we will try to give an overview of their respective contributions to this development, in different fields of work. When it comes to scientific publications, we have relied on the sources at *Zentralblatt MATH* (zbMATH)[35], *ZBL* in further text, for researchers belonging to the earlier period, as it also includes the period before World War II, and *Mathematical Reviews*

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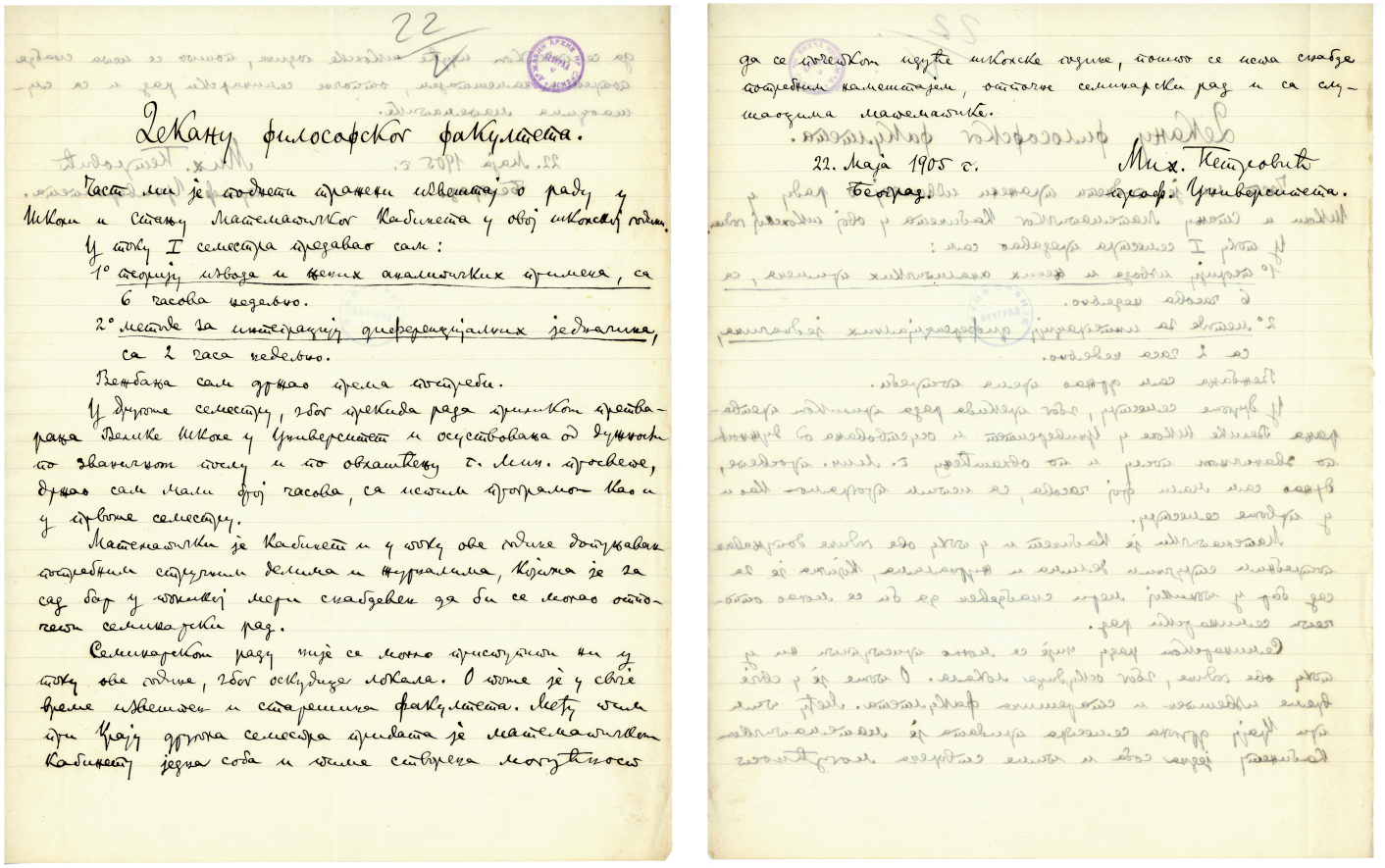
Mathematicians' pictures were drawn by artist Bajo Luković.

(MathSciNet)[36], *MR* in further text, the database of the American Mathematical Society for more recent researchers.

There are 228 Petrović's publications, including 12 books (search under "ai:petrovitch. michel"), in the German referential mathematical journal *Zentralblatt MATH,* which is now edited by the *European Mathematical Society* and the *Heidelberg Academy of Sciences,* and the database of which incorporates, as of 2003, the entries of an earlier similar publication *Jahrbuch über die Fortschritte der Mathematik* (JFM) which existed from 1868 to 1942. His papers referred primarily to ordinary differential equations, complex variable functions, as well as sequences, series and summability. His associates continued to work in these fields of study, but they were evidently encouraged to work on the development of other areas of mathematics as well. This progress spread through the entire territory of former Yugoslavia and outside its borders on the one side, while on the other side it expanded through the introduction of new disciplines emerging in the world. A reverse process was also present, as mathematicians from the other places came to Belgrade, enriching our school of mathematics and adding to the existing knowledge. This refers primarily to the arrival of Anton Bilimović and Nikola Saltikov from Russia and later of Đuro Kurepa from Zagreb. Many of their successors are also to be given credit for the development of the Serbian school of mathematics, but here we will focus only on those who are genealogically connected with Mihailo Petrović.

In addition to choosing their area of work, associates were also given the opportunity to travel abroad in order to expand their knowledge and their views. These visits helped spreading mathematical ideas in the other countries as well. Mihailo Petrović was versatile with regard to his scientific and pedagogical work, establishing of journals, activities in the Serbian Royal Academy, etc. He also enjoyed some other activities such as music, fishing, travelling, writing travelogues and novels, etc.

As we have already said, the Serbian school of mathematics made an important impact in former Yugoslavia and the rest of the world. Since mathematicians gather around these ideas, the expansion of this school has continued to the present day through participation in the work of seminars, conferences and mathematical journals, such as today's *FILOMAT, AADM* and *MATCH Communications in Mathematical and in Computer Chemistry* (*SCI* list journals), as well as other *ESCI* list journals, etc. In recent years, our state universities have improved their ranking in the area of mathematics and they are now among the top 500 in the Shanghai list for mathematics, which is clear evidence of the progress of mathematical sciences in Serbia.



Mihailo Petrović's report on the work of Cabinet of Mathematics in school year of 1904/1905, submitted to Dean of Faculty of Philosophy, 22 May 1905

ROAD TO THE GRAND SCHOOL

Mihailo Petrović was born on 24 April 1868 in the vicinity of Princess Ljubica's Residence and the Cathedral Church, in the Kosančićev Venac neighbourhood on the Sava slope descending to the confluence of the two rivers. When he started elementary school, his father Nikodim died. His mother Milica and her father, Novica Lazarević,[37] took over care of his upbringing and education. Petrović respected his mother's father and preserved a fond memory of him. He often retold his grandfather's sayings and quips, and it is from him that he inherited his love for the ordinary people of the Savamala and Dorćol neighbourhood [Trifunović, 1969, 26].

After finishing elementary school, Mihailo Petrović enrolled in the First Belgrade Gymnasium, then located in the courtyard building of Captain Miša's Edifice. On the occasion of celebrating the 100-year anniversary of the Gymnasium, Petrović said: "If one of the alumni of the First Belgrade Gymnasium told you that he had spent 55 years in Captain Miša's Edifice, without changing during all that time either his vocation or his profession, you would look at that person

with incredulity, wondering if that was even possible. However, not only is it possible, but it was actually the case for the writer of these lines himself, who entered this building at the beginning of school year 1878/79 as first class student of the Gymnasium and left it in school year 1937/38 as retired full-time professor of the University of Belgrade, with only one intermission during his education abroad, after finishing the Grand School which was located in this building, too [Trifunović, 1969, 32]."

In his book, *Chronicles of Life and Work of Mihailo Petrović,* Dragan Trifunović vividly conveys the impressions about education in the Gymnasium of the time. With his friend Pavle Popović, Petrović often commented on the literary abilities of the literature teacher, Andra Nikolić, presented to students by the then Minister of Education, Stojan Novaković. Interestingly, at a later time, Andra Nikolić and Mihailo Petrović were elected the members of the Serbian Royal Academy on the same day, and in 1905, when Petrović was appointed full professor at the University, Andra Nikolić was the Minister of Education. Petrović also had a particular liking for his chemistry teacher, Marko Leko. He studied this subject from a textbook written by Sima Lozanić for Grand School students. His teacher of mathematics was Sreta Stojković, who, as students used to say, was *mathematician by profession, poet by soul.* Mihailo Petrović's friends from the Gymnasium days included Paja Marinković, Jovan Cvijić, Jaša Prodanović, Vladislav Ribnikar, Ljuba Jovanović, and the others. They grew to become a generation that spearheaded the progress of science in Serbia in the decades to come.

Mihailo Petrović enrolled in the Grand School in 1885. A group of subjects was taught by professor Dimitrije Nešić (1836–1904), who heralded the dawn of Serbian mathematics. He was a teaching assistant at the Prince's Lyceum, long-time professor of the Grand School, the first member of the Serbian Learned Society. In 1887, he was elected among the first 16 full members of the Serbian Royal Academy. At that time, Josif Pančić, Dimitrije Nešić, Ljubomir Klerić and Jovan Žujović were in the Academy of Natural Sciences.

Professor Nešić's personality was such that he was able to convey to his students love for the subject; he was known for clarity of exposition; he directed his students' attention and taught them to be able to tell the important from the unimportant; he identified himself with the science he was teaching [Trifunović, 1996, 19]. Nešić began his studies at the Lyceum in Belgrade, then continued his education at the Vienna Institute of Technology and finally at the Polytechnic Institute in Karlsruhe. He was a true devotee of the temple of education, a humane, noble man, a man of *angelic soul.* He was considered to be an ideal man.

In 1873, the Faculty of Philosophy was divided into two departments: Department of Philology and History and the Department of Sciences and Mathematics. As of 1880, under the Law of the Constitution of the Grand School, it was determined that studies at the Faculty of Philosophy should last for four years. New subjects were introduced and the curriculum was expanded. In 1887, Bogdan Gavrilović, a young doctor of mathematical sciences was appointed at the Grand School. From this appointment until the end of his life (1947) and, from 1894 onwards together with Mihailo Petrović, he will, in a quiet manner of a scientist and an outstanding organiser, build our higher education and produce several invaluable contributions to the mathematical science.

PARIS SCHOOL OF MATHEMATICS

At the Grand School, Petrović received a general education in natural sciences. He had no speciality, but he did demonstrate an affinity for mathematics. In addition to mathematics, he was also an exceptional student of chemistry, taught by professor Sima Lozanić, mechanics, taught by professor Ljubomir Klerić, and history, taught by professor Srećković. The young Petrović was therefore well prepared for Paris. Paris was, at the time, the centre of scientific Europe. It was the hub of scientific and technological innovation. The mathematical school was particularly strong. A mathematician of this school emanated his universality. Such were, for instance, Henri Poincaré, Picard, Painlevé, Hermite, Darboux and the others.[38]

Thanks to Novica Lazarević, Mihailo Petrović continued his studies in Paris. He saw his grandson off with the following words: "I will see with the government that you get a state scholarship, your job meanwhile is to study." After an application by the minister of education to the French minister of foreign affairs, the preparation and sitting of the demanding entrance examination, Petrović continued his studies at *L'Ecole Normale Supérieure*.

After Dimitrije Danić and Bogdan Gavrilović, in 1893 the scientific environment in Belgrade was enriched by a new doctor of mathematical sciences, Đorđe Petković, who obtained his PhD in Vienna. The Serbian Royal Academy of the time had three mathematicians in its ranks: Dimitrije Nešić and Ljubomir Klerić were full members, while Petar Živković, the headmaster of the Gymnasium, was a corresponding member. Mihailo Petrović defended his doctoral dissertation *On Zeroes and Infinities of the Integral of Algebraic Differential Equations*[39] in Paris in 1894, before a commission made up of renowned professors and leading mathematicians of the time, Charles Hermite, Émille Picard and Paul Painlevé. "This young

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Henri Poincaré

ace from Savamala and Dorćol, a masterful mathematician at the Grand School, the holder of two Orders of St. Sava, the author of appreciated student discussions, lived up to everyone's expectations. In Paris, he was ranked the best together with three other students of the school. Novica's grandson, his success in Paris, his doctoral title and other degrees in mathematical sciences were the talk of the entire town of Belgrade [Trifunović, 1969, 126]."

Mihailo Petrović was one of the best doctoral students of his generation in Paris. Emile Picard included his result on particular uniform integrals of differential equations into his textbook on analysis.

Belgrade Grand School (University of Belgrade) in 1907 (Belgrade City Library, fund F-1–191, inventory number 1593)

MIHAILO PETROVIĆ'S BELGRADE SCHOOL OF MATHEMATICS

In 19th-century Serbia, there were six doctors of mathematical sciences. They were **Dimitrije Danić** (1862–1932), who obtained his PhD in 1885 in Jena, **Bogdan Gavrilović** (1864–1947), who obtained his PhD in 1887 in Budapest, **Vladimir Varićak** (1865–1942), who obtained his PhD in 1891 in Zagreb, **Đorđe Petković** (1862–1934), who obtained his PhD in 1893 in Vienna, **Mihailo Petrović** (1868–1943), who obtained his PhD in 1894 in Paris and **Petar Vukićević** (1862–1941), who obtained his PhD in 1894 in Berlin [Kečkić, 1985, 3–6].

After professor Dimitrije Nešić left to assume state administration duties and his retirement in 1894, a vacancy for a professor of mathematics was announced at the Grand School. Competition was very strong: Dr Mihailo Petrović got 11 votes of the Academic Council of the Grand School, Dr Petar Vukićević – 10 votes and Dr Đorđe Petković – one vote. Mihailo Petrović succeeded Dimitrije Nešić as a professor of mathematics. His lectures were easy to understand, as he tried to maintain a level that was accessible to the listeners. He encouraged independent work in those who wished to expand their knowledge. He was direct, modest and had a cheerful spirit. He managed to bring the harmony of his spiritual qualities into everyday life. He considered scientific work to be the foremost duty of a university professor, as without science there could be no success in teaching and no progress at all.

Mihailo Petrović

He was elected corresponding member of the Serbian Royal Academy in 1897, at the proposal of his professors, now colleagues at the Grand School, Dimitrije Nešić, Sima Lozanić, Jovan Žujović and Ljubomir Klerić. At the proposal of the same group of academicians, he became full member of the Serbian Royal Academy in 1899. In the same period, he was elected member of several foreign scientific societies, and had already published an imposing number of scientific papers.

An important date in the development of education in Serbia was 19 February 1905, when University Law was enacted. The efforts of Grand School professors had finally borne fruit. Serbia got a University. At the proposal of the education minister, by a decree of the King of Serbia, the first eight full professors of University were appointed on 27 February: Ljubomir Jovanović, Dragoljub Pavlović, Milić Radovanović, Jovan Žujović, Andra Stevanović, Jovan Cvijić, Mihailo Petrović and Sima Lozanić.[40]

In 1909, at the proposal of Mihailo Petrović and Jovan Cvijić, the University of Belgrade invited Milutin Milanković, the civil engineer from Vienna, to work as professor of applied mathematics at the Faculty of Philosophy. As a result, Petrović and Milutin Milanković shared not only their office, but also the universal world of mathematics. Petrović taught theoretical mathematics and Milanković – applied mathematics. In 1920, Milanković was elected corresponding, and in 1924, full member of the Serbian Royal Academy.

In 1912, Mladen Berić defended the first doctoral dissertation in mathematical sciences at the University of Belgrade, before a commission made up of Mihailo Petrović and Milutin Milanković. A year later, Sima M. Marković defended his doctoral dissertation before the same commission. The letter which Petrović thereafter sent to the Council of the Faculty of Philosophy, as justification for appointing another lecturer to teach theoretical mathematics, represents the cornerstone of future scientific work at the University and the start of creation of the mathematical school.

Parts of this letter outline actual intentions to develop mathematical sciences: “By working alone, the teacher could only teach the indispensable elements, without having the opportunity to move on and introduce to listeners some of the

branches of Mathematics that are developing today, that containing topical problems, that are, therefore, the most fitted for independent work, and that cannot be embarked upon until elementary things are mastered. However, it is precisely in these areas of Mathematics that we now have no time to embark upon that the system of today's scientific work is reflected and it would be of utmost benefit to introduce listeners into independent work on this terrain precisely, which in a science like Mathematics, where each part is the foundation for the next, is impossible to realize with the programme of work which has so far been applied."

After World War I, there was growing demand for teaching staff as the number of young people attending schools and of university students increased. Petrović dedicated his attention to this issue as well, aware as he was that a teacher of mathematics could, through his approach, either help students acquire love for the subject or turn it into a genuine "scare". For this reason, he advocated in the Council that it would be very useful for teachers to be well grounded in mathematical didactics and the methodology of teaching mathematics. He sent letters and requests to the authorities asking for an increase in the number of University professors in order to enable the spreading of scientific knowledge but also the introduction of pedagogical sciences to students so as to improve the quality of teaching. The lectures would thereby become clearer, more simplified and delivered in a systematized manner. This contributed to a greater interest in mathematics.

The period from 1921 onwards was significant for the development of mathematics at the University of Belgrade as two distinguished names of the mathematical science arrived in Belgrade: Anton Bilimović and Nikola N. Saltikov. There was also the younger generation: Vjačeslav Žardecki, Tadija Pejović, who obtained his PhD in 1923, and Radivoje Kašanin, who obtained his PhD in 1924 (both before a commission comprising Petrović and Bilimović), Jovan Karamata, who obtained his PhD in 1926 (the commission comprised Petrović, Bilimović, Saltikov), Miloš Radojičić, who obtained his PhD in 1928 before a commission comprising Petrović and Saltikov.

The year 1931 marked the start of publication of the journal *Matematički list za srednju školu (Mathematical Journal for Secondary School Students)* in Belgrade, edited by professors Radivoje Kašanin, Vojislav V. Mišković and Jovan Karamata. In the same year, together with his colleagues at the Faculty of Philosophy, Mihailo Petrović set up the famous mathematical journal entitled *Publications Mathématiques de l'Université de Belgrade*. Dragoslav S. Mitrinović obtained his PhD in 1933, before a commission comprising Petrović, Saltikov and Pejović, while Danilo Mihnjević and Konstantin Orlov obtained their PhDs before the same commission a year later. Petar Muzen obtained his PhD before the same commission in 1937, and Dragoljub Marković – in 1938.

Radivoje Kašanin and Jovan Karamata, both doctoral students of Mihailo Petrović, later became full members of the SASA, while Miloš Radojčić was a corresponding member. From the table above, it is evident that Mihailo Petrović's disciples for the most part wrote their doctoral dissertations in the area of differential equations.[41] Through their teaching and scientific work at the University of Belgrade, as well as their activities at the Seminar set up by Petrović and the publication of scientific papers in Belgrade and mathematical journals worldwide, this group of mathematicians, Petrović's students, made up the core of the mathematical school called the Belgrade School of Mathematics.

	Name and surname	year	Name of doctoral dissertation	Commission
1.	Mladen Berić	1912	Figurative Polygons of First-order Differential Equations and their Relation with the Properties of Integrals	Mihailo Petrović Milutin Milanković
2.	Sima Marković	1913	General Riccati Equation of the First Order	Mihailo Petrović Milutin Milanković
3.	Tadija Pejović (17 doctoral students)	1923	New Cases of Integrability of an Important Differential Equation of the First Order	Mihailo Petrović Anton Bilimović
4.	Radivoje Kašanin (two doctoral students)	1924	On Analytic Forms of Multiform Functions	Mihailo Petrović Anton Bilimović
5.	Jovan Karamata (12 doctoral students)	1926	On Certain Limits Similar to Definite Integrals	Mihailo Petrović Anton Bilimović Nikola Saltikov
6.	Miloš Radojčić	1928	Analytic Functions Expressed in Terms of Convergent Series of Algebraic Functions	Mihailo Petrović Nikola Saltikov
7.	Dragoslav Mitrinović (33 doctoral students)	1933	Investigations of an Important Differential Equation of the First Order	Mihailo Petrović Nikola Saltikov Tadija Pejović
8.	Danilo Mihnjević	1934	Structure of Partial Equations with Given Characteristic Integrals	Mihailo Petrović Nikola Saltikov Tadija Pejović
9.	Konstantin Orlov (nine doctoral students)	1934	Arithmetic and Analytic Applications of Mathematical Spectrums	Mihailo Petrović Nikola Saltikov Tadija Pejović
10.	Petar Muzen	1934	On Bases of Continuous Functions	Mihailo Petrović Nikola Saltikov Tadija Pejović
11.	Dragoljub Marković (one doctoral student)	1938	Limits of Roots of Algebraic Equations	Mihailo Petrović Nikola Saltikov Tadija Pejović

Mathematicians who obtained their PhD under the supervision of Mihailo Petrović

When Mihailo Petrović was awarded the title of honorary doctor of sciences of the University of Belgrade in 1939, he was referred to as the "engineer of mathematical sciences in our country, while the degree he received specified that his major achievement was the *Belgrade School of Mathematics,* whereby he was duly credited for his outstanding scientific work in all areas of mathematical sciences and the creation of a school of mathematics at the University of Belgrade [Trifunović, 1969, 412]."

The focus of Mihailo Petrović's work were differential equations and the function theory. It is interesting to note that the bulk of Mihailo Petrović's papers were published in Paris, but also that his papers were presented at the Paris Academy by great mathematicians of the time. Such approach to Petrović mirrored the approach of these great men of science to work in general. It was in fact an ode to the act of creation. Or as Newton said: "Hard work and dedication to learning are the highest hopes of humanity". Petrović applied the same style. He published and presented the papers of his doctoral students at the Serbian Royal Academy. This approach encouraged his students to improve and create further. Scientific work in Belgrade was therefore based on grand ethical postulates.

Petrović's qualitative analysis of solutions of differential equations was interesting and inspiring, particularly for that time. In this qualitative analysis, he did not look for the solution to the differential equation, which at the times can or cannot be found, while the quest for it is very complicated, but he rather tried to provide as much information as possible about the nature of the solution based on the properties of the equation itself.

Mihailo Petrović investigated the Riccati differential equation $y' = y^2 + f(x)$ and its generalization $(y')^2 = y^2 + f(x)$. Inspired by the papers of Mihailo Petrović, at a later time Sima Marković (also in his doctoral dissertation), Tadija Pejović (in his doctoral dissertation and a number of papers), Dragoslav Mitrinović (in his doctoral dissertation and in around 25 papers) and Milorad Bertolino (in about ten papers) studied the above differential equations. In Petrović's papers, the qualitative analysis of solutions to differential equations also included the so-called qualitative first integral, the nature of the solution and the properties, the number of zeroes and their distance, etc. He also ventured into the theory of numbers, the theory of polynomials, complex analysis, applications in numerical mathematics or chemistry.

We will quote an exceptional description of mathematics and Mihailo Petrović's place in it from his *Collected Works,* published by the Institute for Textbook Publishing: "Mathematics is a strange world in our universe. It is not known who its creator is – God or man. Therefore, it is not known whether a mathematician discovers the entities and their mutual relations in it or whether he himself creates them. Either way, it is both wonderful and exciting to be in that world especially to those who know how to discover it and enjoy in it. Mihailo Petrović spent many beautiful hours of his rich and diverse life in the mathematical world, discovering or creating, and he left a part of himself in it. This world is unique. Its classification into individual areas, which often overlap, is more administrative in nature. Mathematician Petrović was versatile.

He trod his roads, in pursuit of his ideas and visions. His papers, like memories from these travels, cannot always be classified into a single area. What is more important are the ideas that he had and the way in which he realised them [Aranđelović, 1999, 281]."

Petrović had a developed geometrical view on mathematics, which is particularly conductive to intuition or a sense of result. And whereas intuition can mislead, the element of creation is important for a creator. For instance, Petrović proved the following inequality:

$$\left|\sum_{k=1}^{n} a_k\right| \geq \cos\frac{\lambda}{2}\sum_{k=1}^{n}|a_k|$$

on condition that complex numbers $a_1, a_2, \ldots, a_n$ are located in an angle with vortex at the beginning and measure $\lambda < \pi$, which is symmetrical with respect to the real axis.

Inequalities are one of the fundamentals of mathematical analysis. Well known is Petrović's inequality for convex functions at $[0, a)$, $(a>0)$ [Publ. Math. Univ. Belgrade 1 (1932), 149–156]

$$f(x_1)+f(x_2)+\cdots+f(x_n) \leq f(x_1+x_2+\cdots+x_n)+(n-1)f(0),$$

for which he is most often cited in literature. Even before, he considered this inequality to represent only a narrower class of functions (functions that can be represented by a power series with positive coefficients). Petrović's inequality has been used and generalised in different directions by a large number of mathematicians in the world!

Petrović also studied generalisations of some of Stiltjes formulas, but also many other issues relating to mathematical analysis. In the afterword to book 3 of *Collected Works* of Mihailo Petrović, a fine conclusion is drawn: "New mathematical structures are born out of a wish to cover everything that has been discovered so far; Petrović thus trod his own path, in pursuit of his own visions. He opened many new problems. He solved them impatiently, often not seeing them through. He generously left this for others to do. He was in constant search of a higher connection of mathematics and life, or at least a part of it. Because, finally, mathematics is life, too". [Aranđelović, 1999, 296].

In addition to differential equations, Mihailo Petrović also wrote some papers on the analytic function theory and inequalities, but his writings also included phenomenology, travelogues, etc. Like his professor Henri Poincaré, he wished to express his universality and thus be elevated to higher levels of knowledge. This is also the avenue for learning mathematics. Petrović considered metaphors and allegories subjective, but still belonging to a lawful form of human knowledge, spirit and consciousness. They had a deep sense and a deep root in human consciousness and corresponded to an instinctive and irresistible need of the spirit and the consciousness. They even tied themselves to certain facts and represented a specific expression of existence of these particulars. The methodology of natural sciences included, among other things, anticipation, analogical mapping, modelling and represented the development of thought in science, art and life in general.

A MAN OF INTELLECTUAL VIRTUE

This is how Milutin Milanković described his colleague Mihailo Petrović: "Petrović spiralled upwards to reach the peak of the pyramid of exact sciences already as a young man, when he left for Paris, in a bid to quench his thirst, tapping into the most abundant spring of mathematical knowledge, and then, imbued by the spirit of this science, he reached those boundaries of science where its new unsearched areas began. Already at the very start, he went past these boundaries with the doctoral dissertation which he defended in 1894 at Paris University before a commission comprising three most celebrated mathematicians of the time, Hermite, Picard and Painlevé. That year Mihailo Petrović returned to Belgrade as an accomplished and renowned scientist, and, appointed full professor at the Grand School, he also came to occupy the position of the greatest mathematician. This is the position he kept until his death – for one half of a century [Trifunović, 1969, 428]." Milanković spoke of Petrović's personality, of a man who received five Orders of St. Sava. Of how Petrović brought and sowed the seed of the mathematical science in Serbia and created there a sprouting ground of mathematical knowledge. Petrović's work was not limited to the education of secondary school teachers of mathematics only, but he managed to create scientists out of talented students and to prepare them for independent work.

"In terms of scientific work, he took precedence over us all. Since 1894, when his first study was published in the reports of the French Academy of Sciences, he published two and a half hundred scientific papers, of which twelve represent self-standing scientific works... He diligently compiled this treasure trove for the sake of the science, not for himself, because he never even thought of deriving from it any personal benefit, fame or celebrity. It was one of the most beautiful features of his character and of his entire work... He was a man brimming over with feelings, he knew how to enjoy everything beautiful that life had given to him. He loved company and music, his favourite sport (fishing) and travels... A tranquil, quiet, humanly simple, superhumanly gifted, Petrović was one of the greatest sons of our country [Trifunović, 1969, 429]."

Jovan Karamata also wrote about Petrović: "During his many years of fruitful work, Petrović touched upon almost all areas of mathematics, and though they differ significantly among them, he was guided by the procedures and considered them primarily from a viewpoint of mathematical analysis. [Trifunović, 1969, 362]." These papers cover the areas of algebra, arithmetic, integral calculus, theory of functions, differential equations, mathematical physics, chemistry and general phenomenology. Milan Bogdanović said: "The opus of Mr Mihailo Petrović is to the fullest possible extent contemporary, both with regard to its content and its spiritual orientation that goes hand in hand with both time and the spirit of time."

Tadija Pejović

DEVELOPMENT OF THE SERBIAN SCHOOL OF MATHEMATICS

Almost all doctoral students of Mihailo Petrović studied differential and functional equations, with the exception of Jovan Karamata, Dragoslav Mitrinović and Konstantin Orlov. Karamata studied sequences, series and summability, Mitrinović examined real, complex and special functions, and in particular inequalities, while Orlov engaged in numerical analysis of differential equations. This meant a step forward and the expansion of the mathematical school to include not only other people, but also other areas.

Some of the doctoral students achieved particularly noted results. One of the selected criteria is the number of further doctoral students, where Tadija Pejović, Jovan Karamata, Dragoslav Mitrinović and Konstantin Orlov stand out. Tadija Pejović was also exceptional for his textbook writing, Jovan Karamata wrote a particularly large number of scientific papers, earning him world fame, while Dragoslav Mitrinović stood out for the number of papers and the number of published books and Konstantin Orlov for his pedagogical qualities. All four of them together, and their successors, made an indisputable impact on the development of the Serbian School of Mathematics.

If we look at further development of mathematics along different branches, we will see that a number of areas developed in Serbia. For instance, *differential equations* were the object of study of doctoral students of Tadija Pejović, Dragoslav Mitrinović and Konstantin Orlov; *mathematical analysis* – of doctoral students of Dragoslav Mitrinović, Jovan Karamata and Tadija Pejović; *algebra* – of doctoral students of Tadija Pejović and Dragoslav Mitrinović; *numerical analysis* – of doctoral students of Dragoslav Mitrinović and Konstantin Orlov, etc.

The start of scientific and pedagogical work of Tadija Pejović[42] (Drača, 1892 – Belgrade, 1982) is marked by his doctoral dissertation *New Cases of Integrability of an Important Differential Equation*, which was completed in 1922 and defended on 6 February 1923 before a commission comprising Mihailo Petrović, Milutin Milanković, Anton Bilimović and Vladimir Petković (dean of the Faculty of Philosophy). The subject of the dissertation was the generalised Riccati differential equation $(y')^2+y^2 = H(x)$ which had

even before been the subject of consideration by Apelle, Elliot, Mihailo Petrović, Liouville, and which was later studied in Belgrade by a number of mathematicians (D. Mitrinović, M. Bertolino, Lj. Protić). In his first scientific paper, Tadija Pejović also dealt with the problem of invariants of this same differential equation, which he will continue to examine in several later papers. When on 1 March 1948 the Society of Mathematicians and Physicists of Serbia was founded, Tadija Pejović was its first president (1948–1952). He taught at the Faculty of Philosophy and later at the Faculty of Sciences and Mathematics at the University of Belgrade. He was also the dean of the Faculty of Sciences and Mathematics.

In his pedagogical work, Tadija Pejović was particularly known for the publication of textbooks. He published *Mathematical Analysis* in five books and *Differential Equations* in three books, with the third book dedicated to the existence of solutions. The number of pages in the above textbooks is over two thousand, which is unusual in mathematical publications. All of the above books were reprinted, some for more than ten times (for instance, *Analysis I* was printed thirteen times).[43] He had 17 doctoral students including Vojin Dajović (1956), Ernest Stipanić (1957), Milorad Bertolino (1961), Nedeljko Parezanović (1962), Slaviša Prešić (1963), Zoran Ivković (1963), Milosav Marjanović (1964), Zoran Pop-Stojanović (1964), Petar Todorović (1964), Rade Dacić (1965) and Časlav Đaja (1967).

Vojin Dajović defended his doctoral dissertation *On the Existence of Limit Values of Some Classes of Analytic Functions* in 1956 at the Faculty of Sciences and Mathematics of the University of Belgrade, supervised by Tadija Pejović. After receiving his PhD, he developed **Complex Analysis** and continued the tradition of work with young and gifted students. According to MR, Dajović had 16 papers in the area of complex functions. He participated in the reform of the teaching of mathematics and physics at all levels and advocated a general promotion of mathematics as a fundamental science and one of the most important subjects at all levels of education.

Vojin Dajović wished to see a development of the impact of mathematics and mathematicians on the improvement of the educational system and the social reality at large. In particular, his contribution is reflected in his ability to recognise and foster gifted young mathematicians; he was the initiator of the idea and the founder of the Mathematical Gymnasium in Belgrade. He participated in the establishing of the Society of Mathematicians and Physicists of Serbia and in the founding of the Association of Mathematicians and Physicists of Yugoslavia. He successfully organised congresses of mathematicians. In particular, he advocated the introduction of the subject Methodology of Mathematics. At the Belgrade Faculty of Mathematics, doctoral studies of Methodology of Teaching Mathematics were introduced. Continuing Dajović's vision, the Faculty of Mathematics has organised symposiums entitled *Mathematics and Its Applications* each year since 2008.

Vojin Dajović had nine doctoral students, including Miodrag Mateljević with 82 publications (MR), Miroljub Jevtić with 79 papers (MR), Miloje Rajović with 54 papers (MR), Dušan Georgijević with 25 papers (MR) and Mioljub Nikić with 18 papers (MR). The most successful is Miodrag Mateljević, whose field of work is the geometric theory of functions and harmonic analysis and who has eight doctoral students including Vladimir Marković (the member of the

British Royal Society) with 43 papers (MR) and David Kalaj with 83 papers (MR), who works at the University of Montenegro. Mateljević chairs the Seminar for Mathematical Analysis which seeks to promote the geometric theory of functions.

Slaviša Prešić[44] obtained his PhD in 1963, with a doctoral dissertation entitled *A Contribution to the Theory of Algebraic Structures.* He published around 50 publications (MR). He began his scientific career with Dragoslav Mitrinović, with whom he wrote several joint papers in the area of difference and functional equations, for which he is recognisable. Slaviša Prešić was a very fruitful mathematician and he spearheaded further development of Algebra and Mathematical Logic in Serbia, but he also wrote papers in the other areas such as numerical analysis, geometry of polynomials and theoretical programming. He had 14 doctoral students who continued to work in these areas: Janez Ušan with 105 papers (MR), Žarko Mijajlović and Gradimir Vojvodić with 50 papers each (MR), Dragić Banković with 48 papers (MR), Svetozar Milić with 29 papers (MR), etc. With regard to the number of doctoral students, the most successful were Žarko Mijajlović with 12 and Svetozar Milić with seven doctoral students; their doctoral students for the most part continued and built their university careers in Novi Sad: Zoran Stojaković with 60 papers (MR) in Algebra and Combinatorics, Stojan Bogdanović with 149 papers (MR) in the Theory of Semigroups and Automata Theory, Siniša Crvenković with 64 papers (MR), Branimir Šešelja with 119 papers (MR) and Andreja Tepavčević with 89 papers (MR), published mostly in the area of Algebra and Mathematical Logic. Ušan and Vojvodić built their careers at the University of Novi Sad, and Banković at the University of Kragujevac. In late 1980s, Bogdanović moved to Niš where he organised a school in these areas. Among six of his doctoral students, the most successful is Miroslav Ćirić, now a professor at the Faculty of Sciences and Mathematics in Niš who has so far had nine doctoral students already and has published 127 papers (MR) in the area of Theory of Semigroups, Automata Theory and Theoretical Computer Science. The doctoral students of Žarko Mijajlović continued their university careers elsewhere – Slobodan Vujošević at the University of Montenegro in Podgorica, Miodrag Rašković at the University of Kragujevac, Milan Grulović at the University of Novi Sad, etc.

Zoran Ivković had 51 publications (MR) on probability theory and stochastic processes. He had 12 doctoral students, including Jovan Mališić with five doctoral students and Svetlana Janković with 52 papers (MR) in the area of probability theory, stochastic processes and differential equations. Among Mališić's doctoral students are Pavle Mladenović with 28 papers (MR) and 10 doctoral students, Tibor Pogány with 160 papers (MR) in the area of probability theory, stochastic processes and special functions (employed at the Rijeka University) and Biljana Popović with 31 papers (MR) and two doctoral students. Svetlana Janković and Biljana Popović work at the Faculty of Sciences and Mathematics in Niš.

Milosav Marjanović has 42 papers (MR) in the area of general topology, functional analysis, convex and discrete geometry and real analysis. He is the founder of topology in Serbia. He has also dealt with issues relating to the teaching of mathematics. He had eight doctoral students, including Rade Živaljević with 60 papers (MR) and Siniša Vrećica with 29 papers (MR). Žarko Mijajlović is also specified as one of the supervisors of Rade Živaljević.

Ernest Stipanić published 29 papers (MR). He studied the history of mathematics, summability of series and mathematical logic. Milorad Bertolino published 36 papers (MR) in the area of ordinary differential equations and the history of mathematics. Nedeljko Parezanović was engaged in the introduction of informatics and computing and had six doctoral students. Zoran Pop-Stojanović was at the University of Florida, USA, since 1965. He published 39 papers (MR) in the area of Probability and Stochastic Processes. From his retirement until his death (2011) he visited Serbia and organised stochastic seminars. Rade Dacić published 32 papers (MR) in the area of Algebra and General Topology. Časlav Đaja published 21 papers (MR). His doctoral student Miloš Čanak published 58 papers (MR) in the area of Complex Functions and Differential Equations.

Jovan Karamata and Dragoslav Mitrinović made a special contribution to the development of **mathematical analysis** in Serbia. In 1926, Jovan Karamata (Zagreb, 1903 – Geneva, 1967) defended his dissertation *On Certain Limits Similar to Definite Integrals* before a commission made up of Mihailo Petrović, Anton Bilimović and Nikola Saltikov. He published 160 scientific papers (according to ZBL) in the following areas: Sequences, Series and Summability, Theory of Numbers, Fourier Analysis and other areas of mathematical analysis. Most joint papers he published together with Miodrag Tomić (6) and Bogdan Bajšanski (4).

Karamata's approach to the introduction of new subjects and the manner of their exposition was quite radical, which sometimes even made students protest. New scientific concepts and new theories are a difficult thing, and already accustomed opinion resists new ideas. At that time, not much discussion took place with regard to whether the faculty was a school for educating teachers or primarily a scientific institution. Karamata thought that science was the only goal of the faculty, so he attuned his lectures, even introductory ones, to this principle. Hence there is no wonder that the majority of students did not understand him. In order to have more time for contemplating mathematics and writing papers, he had the custom to hold all his lectures in a single day. A number of completely different courses: Elementary Algebra, Higher Algebra, Introduction to Analysis, Theory of Sequences and Series, and Descriptive Geometry – all in quick succession one after another.

Jovan Karamata

During break time, Jovan Karamata's office was packed with students who brought in mathematical problems or asked for advice regarding their seminar papers. During his lectures, he also presented students with problems. Some were so difficult that at first students did not even understand them. There were students who spent days trying to solve some of the problems. Some dedicated all their work to interpreting his lectures. Others, by contrast, stopped attending his classes altogether. As he alone taught a large number of courses in the first year of studies, the number of those who fled mathematics after attending a few of his classes was certainly not small. He prepared some of his lectures in detail and there were moments when students left his classes with radiant faces. The first step when drafting a seminar paper was to study foreign sources, without which it was impossible to even conceive a seminar paper. Students thus began to see that it was not talent but hard work that mattered most.

Jovan Karamata achieved global fame in 1930, when he found a short proof of the Hardy-Littlewood theorem published in his paper *Über die Hardy-Littlewoodsche Umkehrungen des Abelschen Stätigkeitssatzes*. The paper had only two pages, but it caused perturbation in mathematical circles and brought his author immediate world fame. It is very interesting to read Vojislav Marić's testimony of this: "When I was visiting St. Andrews University in Scotland, I was introduced to the renowned mathematician Copson (E. T. Copson) from whose book many from my generation studied the theory of functions of a complex variable. He said: "I have so far heard of only one Yugoslav mathematician, Jovan Karamata. When I was studying under Hardy in the 1930s, I found him one day pacing nervously up and down his office. Without greeting me, and visibly excited, he said: *I got a letter from a young man from Belgrade who claims to have proven the Hardy-Littlewood theorem on two pages only. That is simply impossible*." This Karamata's paper not only brought a new, short and extremely elegant proof of the famous theorem, but also a new method that enabled many future results and applications.[45] Jovan Karamata set the cornerstone of the theory of regularly varying functions and was the author of several important Tauberian theorems. It was soon evident that these functions can successfully be applied in many branches of mathematical analysis and in the probability theory, wherever not only the fact of convergence itself is needed, but also other additional information.[46]

He was elected corresponding member of the Yugoslav Academy of Sciences and Arts in Zagreb, in 1933. The candidate report on Karamata was submitted on 20 February of the same year by academician Vladimir Varićak. In addition to a concise biography and the list of 37 papers published that far, the report also stated: "Though still young, our candidate, Dr Karamata, is already a well-known and reputable person in the mathematical world." He became a corresponding member of the Serbian Royal Academy in 1939, and a full member of the Department of Sciences and Mathematics of the SASA in 1948. Karamata continued his university career in Switzerland.

Jovan Karamata had 12 doctoral students, of whom nine in Belgrade, including Vojislav Avakumović, Miodrag Tomić, Slobodan Aljančić, Ranko Bojanić, Vladeta Vučković, Bogoljub Stanković, Bogdan Bajšanski and others, and three in Geneva, the most famous being Ronald Coifman, who is now a professor at Yale University. He has 170 papers (MR) published in the area of Fourier analysis and in a number of other areas, as well as over 30 doctoral students.

According to *ZBL* data, Vojislav Avakumović had 44 papers in the area of differential equations, operational calculus, sequences, series and summability, Fourier analysis and other areas of analysis. He had eight doctoral students, of whom six in Germany, including in particular Jochen Brüning with 107 papers (MR) and five doctoral students, Helmut Neunzert with 32 papers (MR) and 40 doctoral students, and Manojlo Maravić with 30 papers (MR) in the area of Fourier analysis, Summability of Series and Differential Equations. Maravić worked at the University of Sarajevo and had four doctoral students.

Miodrag Tomić published 72 papers (MR) in the area of Fourier analysis, Ordinary Differential Equations, Approximations, Special Functions and other areas of analysis. His papers on the geometry of polynomials have been greatly appreciated and highly cited.

Slobodan Aljančić defended his doctoral dissertation *On Asymptotic Expansion of A-Summable Linear Functionals* in the Serbian Academy of Sciences on 10 January 1953 before a commission made up of Jovan Karamata, Milutin Milanković, Vojislav V. Mišković, Radivoje Kašanin and Miodrag Tomić. He was elected corresponding member of the SASA in 1961, and full member of the SASA in 1968. He published 50 papers (MR), the majority in the area of Fourier analysis, sequences, series and summability and approximation theory, measure and integration theory, and special functions. In the 1957–1966 period, he published several auxiliary textbooks in the complex function theory, real functions, introduction to functional analysis and measures and integrations. He then published his most famous and influential textbook *Introduction to Real and Functional Analysis* (Belgrade, 1968), which was renewed in the next three editions. Aljančić's book, which is an excellent textbook from the pedagogical point of view as well, made an impact on the education of our mathematicians and represents a qualitative leap relative to standard courses in analysis. Slobodan Aljančić had 13 doctoral students, among whom Milan Tasković and Dušan Adamović were the most successful. Tasković published 92, and Adamović 32 papers (MR) in the area of mathematical analysis.

Ranko Bojanić and Bogdan Bajšanski continued their careers in the USA at Ohio State University, and Vladeta Vučković at the University of Notre Dame in Indiana, and they were very successful. Ranko Bojanić had 67 publications (MR) in the area of Approximation Theory, Fourier Analysis, Theory of Numbers, Partial Differential Equations, etc. One of his most successful doctoral students, and there were nine of them, is Ronald DeVore, who today is member of the American National Academy of Sciences and has 165 papers (MR) and seven doctoral students. Vladeta Vučković published 33 papers (MR) in the area of Mathematical Logic and Summability of Series. Bogdan Bajšanski published 25 papers (MR) in the area of Approximation Theory, Fourier Analysis and Summability of Series. He was more committed to teaching and had 11 doctoral students. In addition to Karamata, one of Bajšanski's supervisors was also Nikola Saltikov (Nikola Saltikov who in turn was the disciple of Vladimir A. Steklov and Aleksandr M. Lyapunov) who had five doctoral students, including Časlav Stanojević who continued his career in America. Stanojević made an outstanding contribution to the development of Serbian and Yugoslav mathematics by organising during the 1980s the famous *Kupari Conferences*, to which he gathered the most renowned global mathematicians of the time in the area of

analysis, both from the East (Sergey Mikhailovich Nikolsky (Сергей Михайлович Никольский), Oleg Vladimirovich Besov (Олег Владимирович Бесов), Sergey Borisovich Stechkin (Сергей Борисович Стечкин), Boris Sergeevich Kashin (Борис Сергеевич Кашин), Sergey Aleksandrovich Telyakovsky (Сергей Александрович Теляковский...)) and from the West (Walter Rudin, Ronald A. DeVore, Richard Askey...), as well as our mathematicians, especially of the younger generation [Milovanović, 2013, 33–40].

The founder of Mathematical Analysis and Functional Analysis at the University of Novi Sad was Bogoljub Stanković. In addition to Karamata, Avakumović was also one of his supervisors. Stanković had ten doctoral students, including Olga Hadžić, Danica Nikolić-Despotović, Endre Pap, Stevan Pilipović, Dragoslav Herceg, Arpad Takači, Đurđica Takači, and others. Stanković published 163 scientific papers (MR), mostly in the area of Functional Analysis, Integral Transformations, Operational Calculus, Ordinary, Partial and Integral Equations, Complex and Special Functions, etc. He published the largest number of papers with Teodor Atanacković – 23 and Stevan Pilipović – 19.

Olga Hadžić published 160 papers (MR) mostly in the area of Operator Theory, General Topology, Differential Equations and Theory of Probability and Stochastic Processes, Danica Nikolić-Despotović published 34 papers (MR) in the area of Operational Calculus and Functional Analysis, while Edre Pap published 202 papers (MR), mostly in the area of Measures and Integrations and Functional Analysis, and had seven doctoral students. Olga Hadžić had four doctoral students, engaging primarily in General Topology, Operator Theory and Probability Theory and Stochastic Processes. They include: Mila Stojaković with 47 papers (MR), Ljiljana Gajić with 57 papers (MR) and Zoran D. Mitrović with 35 papers (MR), who works at the University of Banja Luka.

Stanković's most successful doctoral student and successor is Stevan Pilipović, who has so far published 352 papers (MR) in the area of Functional Analysis, Partial Differential Equations, Operator Theory and other areas of analysis, as well as several books and monographs. The famous publisher *John Wiley & Sons*, published in 2014 two monographs by T. M. Atanacković, S. Pilipović, B. Stanković and D. Zorica under a general title *Fractional calculus with applications in mechanics* and sub-titles *Wave propagation, impact and variational principles* and *Vibrations and diffusion processes*. Pilipović continued the tradition of work with young associates and he supervised 30 doctors of sciences. Dragoslav Herceg published 114 papers (MR) in the area of Numerical Analysis and had 10 doctoral students. Arpad Takači published 69 papers (MR) in the area of Functional Analysis, Integral Transformations, Operational Calculus and Differential Equations. Đurđica Takači had five doctoral students and published 48 papers (MR) in the area of Integral Transformations, Operational Calculus, Differential Equations and Numerical Analysis. Stevan Pilipović's doctoral students include: Mirjana Stojanović with 83 papers (MR), who unfortunately passed away prematurely, Marko Nedeljkov with 43 papers (MR) in the area of Partial Differential Equations, Mirko Kostić with 90 papers (MR) in the Operator Theory and Differential and Integral Equations, Nenad Teofanov with 37 papers (MR) in the area of Functional Analysis, Fourier Analysis and Operator Theory, etc.

The doctoral students of Dragoslav Herceg engaged in numerical analysis, linear algebra and optimisation: Ljiljana Cvetković has so far had 87 papers (MR) and three doctoral students, Nataša Krejić – 51 papers (MR) and seven doctoral students, Zorana Lužanin – 18 papers (MR) and four doctoral students, etc.

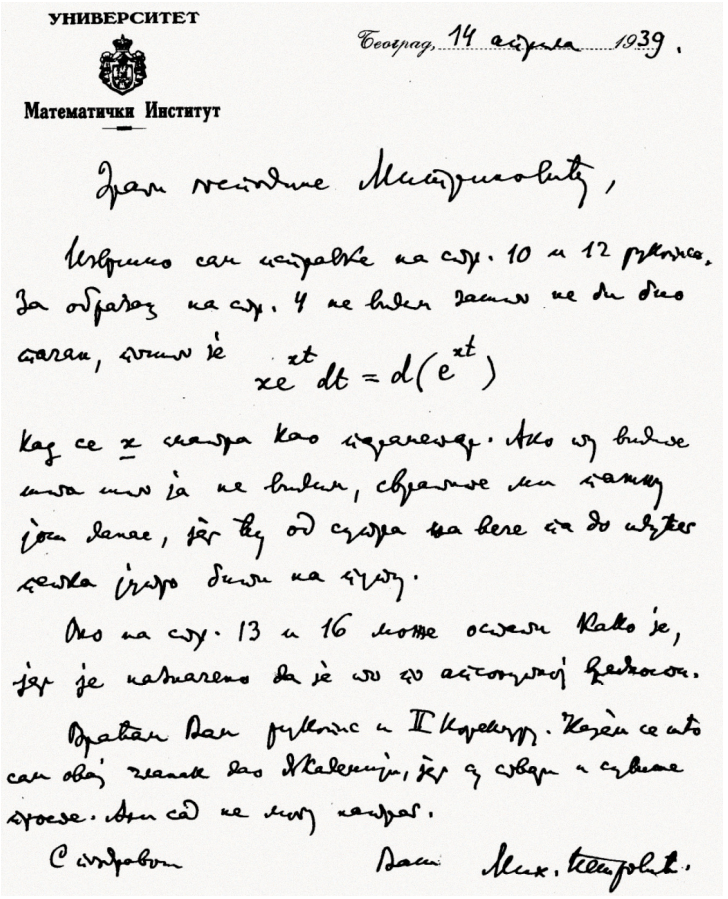
An exceptional student of Mihailo Petrović, Dragoslav Mitrinović (Smederevo, 1908 – Belgrade, 1995) obtained his PhD on 24 October 1933 in the area of differential equations, with a doctoral dissertation entitled *Investigations of an Important Differential Equation of the First Order,* before a commission made up of Mihailo Petrović, Nikola Saltikov and Tadija Pejović. From 1946 until his retirement in 1978, he worked as university professor in Skopje and Belgrade. He was the member of the Macedonian Academy of Sciences and Arts since 1991. The bibliography of Dragoslav Mitrinović has a total of 275 scientific papers and he has been cited 2310 times (MR). In addition, he published 30 other professional papers, 17 monographs, 35 textbooks and 12 other books. He published his papers in the journals at home and abroad, while his books and monographs were published by publishers in the country but also by the world famous publishers. He wrote mostly together with Pečarić, Vasić, Đoković and Kečkić. His main scientific areas included differential equations, functional equations, inequality theory, complex variable functions, special functions and a range of other areas of mathematical analysis.

Speaking of his prolific writing production, Radosav Đorđević accurately noted: "All this, without reprints, amounts to over 30,000 pages. The entire working life of Dragoslav Mitrinović lasted from 1931 until 1994, which is a total of 64 years, including 17 leap years. As this makes up close to 25,000 days, together with all religious and different state holidays, it is easy to conclude that Dragoslav Mitrinović, during 64 years, wrote on average more than one printed page per day, or even three if we take into account the reprinted editions. Or still more, if we exclude the five years of war [Milovanović, 2000a]."

He did not like to travel much, but he always returned from Paris satisfied because during his stay he had collected enough material for further research, for himself and for almost each one of his associates, in the libraries of the Institut Henri Poincaré and the École Normale Supérieure. By government order, after World War II, he was designated to set up the department of mathematics in Skopje. He was the supervisor of Blagoje Popov, the first doctor of sciences in Macedonia. He transferred there the school of differential equations which has been maintained to present day in Macedonia.

Upon his return to Belgrade, he became professor at the School of Electrical Engineering and head of the department of mathematics. He expanded the area of differential equations to include functional equations as well. Mitrinović set up three mathematical journals in Yugoslavia and initiated the publishing of several mathematical editions in the country and abroad. He established a school globally known as the Belgrade School of Functional Equations.

The most important Mitrinović's works were certainly on inequalities in the area of Mathematical Analysis. He considered many important classical inequalities including their generalisations. Especially, let us mention his work on the Steffensen inequality from 1969, as well as a joint paper with P.M. Vasić on an integral inequality ascribed to Wirtinger. In 1974, Mitrinović



УНИВЕРСИТЕТ

Математички Институт

Београд, 14 априла 1939.

A letter from Mihailo Petrović to Dragoslav Mitrinović

and Vasić published one important paper about the history, variations and generalisations of the famous Chebyshev's inequality, and the question of some priorities relating to this important inequality. As far back as in 1965, Mitrinović published the book *Inequalities* on 240 pages; it was published by Naučna knjiga, Belgrade, as part of an edition entitled: "Mathematical Methods in Physics and Tehcnics." Five years later, in 1970, a grandiose work appeared – *Analytic Inequalities* published by *Springer Verlag* (Berlin – Heidelberg – New York).

Professor P. S. Bullen (University of British Columbia, Vancouver, Canada) wrote: "During his long and active life professor Mitrinovic not only did much original work in various fields, although mainly in inequalities. In addition he became famous for research into the obscure origins of many famous results. However his most abiding contribution are three. The famous book, done with the collaboration of professor Vasic, "Analytic Inequalities". It is, after the classic "Inequalities" by Hardy, Littlewood and P6lya, the most referred to book in the field of inequalities. He founded the journal *Publikacije Elektrotehnickog Fakulteta Univerziteta u*

Beogradu, serija Matematika i Fizika, an essential tool for working in the field of inequalities, and the almost complete run that I have is one of my most valuable possessions. Finally there are many students professor Mitrinovic brought along and who are now carrying on his work all over the world. It is no exaggeration to say that they are keeping him alive, and will continue to do so for many years to come."

The monograph *Analytic Inequalities* clearly had a very powerful impact both on the development of this area in our country and globally. It is, certainly, one of the most referred to mathematical books. Inequalities appear everywhere and have an important role in almost all areas of mathematics and other sciences. Dragoslav Mitrinović used to say that *equalities are rare and are almost always an exception, even in everyday life, whereas the inequalities are always met* [Milovanović, 2000b, p. 1–10]. The monograph *Topics in Polynomials: Extremal Problems, Inequalities, Zeros*, written by G.V. Milovanović, D. S. Mitrinović and Th. M. Rassias, and published by the famous publisher *World Scientific* (Singapore – New Jersey – London – Hong Kong), contains important results on the analysis of polynomials and their derivations.

Dragoslav Mitrinović organised a large-scale school of mathematics in Serbia and Macedonia. Under his supervision, 33 doctoral dissertations were written at the Universities in Skopje, Belgrade, Niš, Priština, Kragujevac and Sarajevo.

Blagoj Popov was his first doctoral student in 1952 in the area of Differential Equations and the first to defend a doctoral dissertation in Macedonia, not only in mathematics, but in any other area of science. Popov published 70 papers (MR) in the area of Special Functions, Differential Equations and Operational Calculus. Ivan Bandić was Mitrinović's second doctoral student (1958), also in the area of ordinary Differential Equations, while the third was Lazar Karadžić the same year. Bandić published 47 and Karadžić 30 papers (MR).

Mitrinović's first doctoral student in functional equations, Dragomir Đoković, was one of the most successful; he obtained his PhD in 1963, continued his career at the University of Waterloo and had seven doctoral students. He published, according to MR, 333 publications. His key areas were Combinatorics, Topological and Lie Groups and Algebra, Linear and Multilinear Algebra, Difference and Functional Equations, etc.

Petar M. Vasić obtained his PhD in the area of functional equations in 1963 and published 122 papers (MR) in the area of real functions, inequality theory, functional equations and special functions. Doctoral students in the same area included: Radosav Ž. Đorđević (1966) with 19 papers (MR), Radovan R. Janić (1968) with 53 papers and Ionel Stamate from Romania. Vasić's doctoral student Josip Pečarić, who later worked mostly with Mitrinović, left for Zagreb in late 1980s, where he founded the school of inequalities. He published a vast number of papers in the area of inequality (1193 papers according MR) and has so far had 18 doctoral students.

At the University of Skopje, under Mitrinović's supervision, Ilija Šapkarev obtained his PhD (1964) in the area of differential equations, Dragan Dimitrovski (1968) – in the area of Generalized Analytic Functions and Živko Madevski (1973) – in the area of Inequalities. Šapkarev published 45 (MR) and Dimitrovski 117 papers (MR) in the area of Differential Equations and Complex Variable Functions.

Jovan Kečkić obtained his PhD in 1970 and had 115 papers according to MR. He engaged in the area of ordinary and partial differential equations, difference and functional equations, complex variable functions, linear algebra, inequality theory, special functions, etc.

Dragoš Cvetković obtained his PhD in 1971 in the area of Graph Theory and is the founder of this area in our country. He has so far published 188 papers (MR) on Combinatorics, Graph Theory and Operational Research, including several books. He introduced the spectral graph theory and had three very successful doctoral students. Ivan Gutman works at the Faculty of Sciences and Mathematics in Kragujevac and this is his second doctorate, in addition to one in Chemistry. In the mathematical graph theory, Gutman published 536 papers (MR). Slobodan Simić has so far published 146 papers (MR), and Dragan Stevanović 112 papers (MR) and has already had six doctoral students. Three monographs by Cvetković, P. Rowlinson and Simić were published by the famous publishing house *Cambridge University Press*: *Eigenspaces of Graphs* (1997), *Spectral Generalizations of Line Graphs. On graphs with least eigenvalue-2* (2004) and *An Introduction to the Theory of Graph Spectra* (2010).

Mitrinović's doctoral students at the University of Niš include, among others, Ivan Lacković (1975) with 39 papers in the area of Real Analysis, Gradimir Milovanović (1976) with 292 papers (MR) in the area of Numerical Analysis, Approximation Theory, Special and Orthogonal Functions, and Real and Complex Analysis, Miomir Stanković (1979) with 57 papers (MR) in the area of Special Functions and Numerical Analysis, Miodrag Petković (1980) with 225 papers (MR) in the area of Numerical and Interval Analysis (iterative methods for zeroes of polynomials) and Igor Milovanović (1980) with 110 papers (MR) in the area of Real Functions, Inequality Theory, Graph Theory and Computer Sciences. Lacković's doctoral student at the University of Niš was Ljubiša Kocić (1980) with 78 papers (MR) in the area of Approximation Theory and Numerical Analysis.

Gradimir Milovanović has 13 doctoral students.[47] In addition to the above monograph *Topics in Polynomials: Extremal Problems, Inequalities, Zeros* (*World Scientific*, Singapore – New Jersey – London – Hong Kong, 1994), the famous *Springer* publishing house published the book *Basic Theory and Applications* he wrote with Giuseppe Mastroianni, while many generations of students throughout former Yugoslavia studied from his three-volume textbook *Numerical Analysis* (Naučna knjiga, Beograd, 1985). Milovanović's successful doctoral students include Predrag Stanimirović, with nine doctoral students and 166 papers (MR) in the area of linear algebra, optimization and computer sciences, Ljiljana Petković with 75 papers (MR), Miodrag Spalević with 59 papers (MR), Predrag Rajković with 53 papers (MR), Aleksandar Cvetković with 62 papers (MR) and Marija Stanić with 37 papers (MR) in the area of Numerical Analysis and Approximation Theory, Nenad Cakić with 34 papers (MR) and Gospava Đorđević with 28 papers (MR) in the area of Special Functions and the Theory of Numbers. Today, they are all professors at Universities in Niš, Belgrade and Kragujevac.

Miodrag Petković has so far had eight doctoral students, including Slobodan Tričković with 34 papers (MR) and Jovana Džunić with 25 papers (MR). Petković published a large number of books and monographs, including in particular: *Iterative methods for simultaneous*

Dragoslav S. Mitrinović

Konstantin Orlov

inclusion of polynomial zeros (Lect. Notes Math., Springer, 1989), *Complex interval arithmetic and its applications* (Wiley-VCH, 1998) (together with Lj. Petković), *Multipoint methods for solving nonlinear equations* (Elsevier, 2013) (together with B. Neta, Lj. Petković and J. Džunić).

Finally, the other three successful doctoral students of Mitrinović were: Dušan Slavić (1975) with 29 papers (MR) in Numerical Analysis, Petar Lazov (1977) with 49 papers (MR) in Ordinary Differential Equations and Vlajko Kocić (1981) with 63 papers (MR) in the area of Difference, Functional and Differential Equations, who now works in the USA.

Konstantin Orlov (Ufa, Russia, 1907 – Belgrade, 1985), obtained his PhD in the area of spectrum in 1934, before a commission comprising Mihailo Petrović, Nikola Saltikov and Tadija Pejović. He published, according to MR, 52 papers. His fields of work included Numerical Solution of Differential Equations, Ordinary and Partial Differential Equations, Spectrums and Numerical Analysis. He was also one of the first teachers of programming. At his lectures, he frequently expressed his view: "How much students learn is more important than how much they are taught." He published two monographs, *Finding a General Integral of Partial Equations of the Second Order, That are Not Monge-Amperes,* Serbian Academy of Sciences, Belgrade, 1948, and *Numerical Spectral Arithmetic and Algebra,* Society of Mathematicians of Serbia, 1973 [Zolić, 1998]. Konstantin Orlov had nine doctoral students. They included Mihail Arsenović, Petar Madić, Ljubomir Protić, and the most successful of them – Boško Jovanović with 12 doctoral students. He published 148 papers (MR) in the area of numerical analysis and partial differential equations. His most successful doctoral student is Endre Süli, professor at Oxford University. He published 160 papers (MR) and had 25 doctoral students. Boško Jovanović and Endre Süli published a noted monograph entitled *Analysis of finite difference schemes for linear partial differential equations with generalized solutions* (Springer, London, 2014).

As it is shown previously, Mihailo Petrović transmitted his knowledge to younger generations of mathematicians. He did not restrict their work only to those areas in which he

himself worked, but enabled them to expand their scientific ideas further. Mihailo Petrović was the founder of the Serbian School of Mathematics and the teacher of an entire generation of our mathematicians. Though in late 19th century there were six doctors of sciences in Serbia, the development of our mathematical science began with the appointment of Mihailo Petrović at the Grand School. Scientific work was assigned special value, but also the assurance that it must be evaluated by European norms. At that time, Petrović brought experience and knowledge from Paris, the renowned centre of world mathematics where he received his PhD. The French school of mathematics was at the forefront of mathematical teaching of the time, not only because of its professors but also because of their disciples who spread the knowledge further on in the world. The transmission of this experience to Belgrade meant a lot for our country. This was happening at a time when the Grand School was being transformed into a University. The flourishing of new ideas in Belgrade led to a real progress of science.

It is clear from the above scientific and teaching activities that Petrović's school of mathematics expanded to include the entire Serbia, Macedonia, Zagreb, Sarajevo and some other centres. Many of his students left to work abroad – in America, Germany, Switzerland, etc. Hence this impact was allowed to spread beyond the borders of former Yugoslavia. All the centres set up fruitful, mutually beneficial cooperation with some developed centres abroad. Our mathematicians are often plenary speakers at important international conventions. Mathematicians from the other places came to Serbia, including Anton Bilimović and Nikola Saltikov from Russia and Đuro Kurepa from Zagreb. In Belgrade, Kurepa developed new areas of work. Our researchers, particularly young ones, travelled abroad for further education and acquired new knowledge. Some of them returned to the country and continued here their further scientific activities. A number of prolific mathematicians did not return, but they contributed to the expansion and development of mathematical ideas in global centres and have maintained ties and contacts with colleagues in Serbia.

Finally, it is important to highlight the role of the Mathematical Institute SASA which, throughout this period, and in the past three decades in particular, has, in its own specific way, taken care of the unique mathematical area of Serbia, bringing together mathematicians from all the centres to participate in joint projects grouped by scientific area. In addition to the traditional areas, new contemporary areas of work are introduced, with the involvement of primarily younger associates.

Finally, another novelty in the past years is the Doctoral School at the level of Serbia, endorsed by relevant state institutions from Novi Sad, Niš, Kragujevac, Belgrade and Novi Pazar, including the Mathematical Institute SASA.

The Shanghai list[48] is one of the criteria for assessing our mathematical school, which has evidently risen to a significant and respectable level globally. Four state universities in Serbia (in Belgrade, Novi Sad, Niš and Kragujevac) were included in the prestigious Shanghai list in the area of mathematics, published each year by the Jiao Tong University from Shanghai, and this is one of the most influential lists ranking the best universities in the world. The criteria taken

into account include, among some other things, the number of winners of the Nobel Prize and of Fields medals among university alumni or staff, the number and quality of scientific papers. On the top of the list are Princeton, Paris, Stanford, Oxford, New York, MIT, Cambridge, etc. The above universities in Serbia are ranked among top 500 in the area of mathematics, and this success is based exclusively on the number and quality of scientific papers in referential international journals. This is the result of development of the Serbian School of Mathematics which, as has already been said in the Introduction, does not include only those who are genealogically connected with Mihailo Petrović, though their number is dominant.

In today's Serbia, there are mathematicians committed to teaching, to writing good mathematical textbooks, to producing prestigious scientific papers and publications. Their enthusiasm, their commitment and their work stand witness to future development of the Serbian School of Mathematics.

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